MAP 2302—FALL 2000—COMMENTS ON FIRST EXAM

Partial Credit Policy. In general, to get more than a point or two of partial credit you have to make *significant* progress towards a correct solution of the question that was asked.

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The grade scale for this exam is: A 78–110, B 66–77, C 51–65, D 39-50, E 0–38. (I don't work out B+,C+,D+ ranges till the end of the semester.) It's your *scores* on the exams that eventually get averaged together, not your letter grades, so missing a letter grade by a point is not so serious on a single exam—you can make it up by getting one point over the borderline on your next exam.

If you want to know how your score compared to your classmates', here is the list of scores in descending order:

89,86,85,76,75,74,74,72,71,71,68,68,67,66,64,64,63,
59,57,56,56,52,52,46,44,42,37,32,32,27,25,14,2
(number of students = 33).

The class average was 56%; the median score was 63. The class average on problems 1–4 alone was 68% (54.4/80), which I found surprisingly low. The average score on problem 5 was 11% (2.2/20). Although I expected students to find this a hard problem on the exam, I didn't expect it to be *that* hard. The last two times I've asked questions similar to #5 on exams, the average scores on those questions were 73% and 66% (the 66% was for a harder problem, essentially your problem plus the extra credit combined).

General Comments.

I'm surprised that the class didn't do better, at least on the first four problems, since there was nothing on there that was not seen multiple times in homework and/or in class, or that should have taken very long to do for someone conscientiously doing all the assigned homework. Since many of the problems that people did poorly on were problems that were of types gone over repeatedly in class, my doing still more homework problems in class would not seem to be the answer. The problem seems more to be one of *retention.* This issue is addressed not by how many problems you watch me do in class, but by how many you do on your own.

Although I'm sure some students would like me to do more examples and problems in class than I do, I already spend the majority of class time (probably more than two-thirds) working through examples that I've prepared in my notes and problems that students ask me. Unfortunately there just isn't time to go over more problems in class without eliminating part of the syllabus, which is an unacceptable alternative for several reasons, among them the fact that this course is a prerequisite for several other courses.

If you're having trouble understanding the lectures or doing homework, you can always see me in my office, either by coming to scheduled office hours or by making appointments with me. Don't wait till we're on to next topic or the next exam is drawing near.

I will soon have available a web page with some suggestions on how to take and use your notes more effectively. Meanwhile, here are some other suggestions to help improve your performance.

- Do all the assigned homework problems. Do each problem from start to finish—don't stop when you think you're almost there and that you could complete the problem but don't want to spend the time. Also, don't look at the answer in the back of the book until you've checked the problem yourself. The more you get used to doing homework this way, the faster it will go, and the more confidence you'll develop. I know that many of the problems are repetitive. This is intentional. For basic skills, *repetition aids retention,* probably more than does any other tool. You are able to walk and to write words without any conscious effort because you did these things repetitively in your childhood. If you want to improve at foul shots, you shoot lots of fouls, building "muscle memory". Your mind works the same way.

- When you know you've gotten the wrong answer to a problem, redo it from scratch without looking at your original solution. If you keep getting the same wrong anwer, try to find out from another source (e.g. the book, a classmate, or me) how to get the right answer, and then redo the problem without looking at the solution you've just been shown. Repeat this until you can do the problem correctly.

- Never look up an answer in a solutions manual until you have spent at least an hour *on that problem alone* without getting anywhere. Once you do look at the solutions manual, close the manual after looking at its solution, and then redo the problem from scratch.

- Do all homework on time. Don't let it build up. Read ahead whenever an assignment is short.

- Get into the habit of doing easy consistency checks at the end of a problem (or at the end of each long step). Develop self-monitoring skills through repetition and experience.

- Review your notes daily. If you have time, rewrite them. If questions arise during your review, write them down—don't just put a question mark next to that line of your notes; you'll forget what you were thinking—and see your instructor promptly, during an office hour or scheduled appointment, to get your questions answered.

- Studying for an exam should have at least three components: reviewing your notes, reviewing the reading, and reviewing the homework problems (of which the quiz problems are a small fraction). If you are given an old exam to look at, count that as a fourth component of your study, not a substitute for any of the other three components. If you plan to use the old exam to test your own preparation, try to take the exam under exam conditions (no books, no distractions, working problems from start to finish, same amount of time as you'll get for your exam) to get a sense of the length and difficulty of that exam, and how well you would have been prepared for it. But also remember that there are *many* more topics and types of problems that you're responsible for than can appear on any exam, and therefore that the absence or presence of certain types of questions on the old exam does not imply that the same types of questions will be absent from or present in your exam.

- To be successful in differential equations, you have to have retained what you were taught in algebra, pre-calculus (including trig), and Calculus 1 and 2. You should be able to do integrals more complicated than $\int x^n dx$ accurately and not inordinately slowly. You should have the properties of exponential and log functions at your fingertips. (For a review of exponentials, go to the class home page, click on "other handouts", then click on "Review of exponentials, with practice problems".) Your algebra needs to be accurate; mistakes that seem minor to you can make an enormous difference in the nature of a problem (including whether it's solvable). You should almost never make certain *very* bad mistakes in algebra or in elementary calculus. (I'll give some examples of such mistakes another time. On your exam papers, anything that I've marked with a double-X is in this category.)

- The amount of study time needed for success varies from student to student, and depends on many factors, including prior preparation and innate mathematical aptitude. However, as all entering UF students are told in Preview, in a typical course at UF a typical student should expect to have to spend roughly two hours outside of class for every 50-minute "hour" of class time. That's an *average*, not a maximum: some days you will need to do less, some more; easier courses will take less time, harder courses will take more; better-prepared (or more gifted) students will need less time, worse-prepared students will need more. Also, this estimate is for the time a typical student will need in order to do *satisfactorily* in his or her courses—i.e. to achieve a C or C+—not what may be needed for an A or a B. Most students are capable of doing A or B work *if they put the necessary time into it.* Most students who did not put in the 2-hours-study-per-hour-in-class in their preparatory courses, now have a deficit which can only be remedied by putting in *extra* work now, which of course will add to the amount of study time needed.

- The classes that you work hardest in are the ones in which you learn the most.

Comments on specific exam problems.

Problem 1. In this problem, an *explicit* solution was required for full credit. Remember that any time a DE is written in a form that makes it clear which is the independent variable and which is the dependent variable—for example if the DE is written in “derivative form” as opposed to “differential form”—your final answer should be an *explicit* solution if possible.

Problem 3. (i) This problem was actually part of the homework that was due the Friday before the exam—it’s the exercise at the bottom of the handout “A terrible method for solving exact equations”.

(ii) When solving an exact DE, it is easy to lose sight of your goal: to find a solution of the differential equation. A solution of an exact DE means the same thing as it does for any other ordinary differential equation: a function **of one variable** (the independent variable) which, when plugged into the DE, gives a true statement for all values of the independent variable in the domain of the solution. In general, a function y of a variable x can be defined either explicitly, as in

$$y = x^2, \tag{1}$$

or implicitly, as in

$$x^2 + y^2 = 4. \tag{2}$$

What the equations (1) and (2) have in common is that *they relate x and y to each other*: choosing a value for the independent variable x restricts the values of y for which the equation is a true statement. For example, if we choose $x = 1$ in equation (1), then only for $y = 1$ is the equation a true statement. If we choose $x = 1$ in equation (2), then only for $y = \sqrt{3}$ and $y = -\sqrt{3}$ is the equation a true statement.

In contrast, consider the equation

$$F(x, y) = x^2 + y^2. \tag{3}$$

This equation simply *defines a function of two variables*; it does not relate the variables to each other. If we choose $x = 1$ in equation (3), the equation gives us no restriction whatsoever on y. Thus equation (3) *could not possibly be* the solution of an ordinary differential equation. (Here, *ordinary* means that only ordinary derivatives appear in the DE, as opposed to the *partial* differential equations, which are not the subject of this course. “Ordinary” is not meant in the sense of “garden-variety”.)

Similarly, for the DE in the exam problem, an equation such as

$$F(x, y) = xy - \frac{1}{2} y \cos^2 x + y^3 \tag{4}$$

could not possibly be a solution. However, the equation

$$xy - \frac{1}{2} y \cos^2 x + y^3 = C, \tag{5}$$

where C is an arbitrary constant, *could be* (and, in fact, is) the general solution. If I specify C, so as to single out one solution, and then give a value for x, the values of y are restricted. For example, if we take $C = 7$, then for $x = \pi/2$, y must satisfy the equation

$$\frac{\pi}{2} y + y^3 = 7. \tag{6}$$

The equation (6) has exactly one solution, although with elementary algebra we can't write down the solution explicitly. If we let $f(\pi/2)$ denote this solution, then for x near $\pi/2$, there is one and ony one number, which we'll call $f(x)$, such that $y = f(x)$ is a solution of (6). That's what we mean when we say that an equation determines y implicitly as a function of x.

If your final answer on the exam was equation (4) (or something similar), it's important that you understand why this is *not* a minor mistake. The severity of a mistake can't be measured by the number of pencil strokes needed to correct it. Even though the amount of work needed to turn (4) into (5) is trivial, failing to do that work demonstrates an important misunderstanding of what the purpose of the "exact equations" method is. The purpose is to find a solution to a differential equation, just as if the equation were linear, separable, or anything else.

Once you fully understand that, you'll find that you can't even conceive of writing an equation such as (4) for a final answer to such a problem.

Problem 5. This was basically the homework problem 2.3/35a, which was discussed in detail in class. Although we did only one such problem in class, when going over it I did state more than once that I thought it was a very good problem, and why. Students who asked me whether I would ever put something like that on an exam were told that I'd done it before.

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A quick-and-dirty estimate (**see warning below**) of how you're doing so far is given by translating the information on the syllabus into the formula

$$\text{raw score} = 1000\left(.17\frac{H_1 + H_2 + H_3}{100} + .16\frac{Q_{\text{total}}}{Q_{\max}} + .33\frac{F}{200}\right),$$

where H_1, H_2, H_3 are your midterm exam scores, Q_{total} is the total of all your quiz scores, $Q_{\max}$ is the maximum possible score on all the quizzes put together, and F is your score on the final exam, which will be a 200-point exam. (This doesn't take into account dropping the lowest quiz.) As of today, counting exams not yet given as 0, the maximum possible raw score is 330 (by the end of the semester it will be 1000), and the raw-score cutoffs that this formula gives for various grades are: A 277, B 240, C 199, D 162. **WARNING: Plugging 0's into the formula for exams not yet given can give a very misleading grade-projection,** because doing so gives quiz scores a much higher weight, relative to the exams, than they will have at the end of the course.