Answers to some of the non-book problems

3. $\{y(\ln y - 1) = -x\cos x + \sin x + C\}$. You can't solve explicitly for $y(x)$ in this example.

Writing "$y\ln y - y$" is just as good as writing "$y(\ln y - 1)$"; neither is clearly a simpler expression than the other.

I would not penalize you for omitting the curly braces in answers like this.

4. $\left\{e^{2y}(\frac{1}{2}y - \frac{1}{4}) = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C\right\}$. You can't solve explicitly for $y(x)$ in this example.

I might penalize you a *small* amount for writing "$\ln|1+x^2|$" instead of $\ln(1+x^2)$ in a final answer, since this is an instance of failing to simplify. (Absolute-value symbols around a positive quantity are not needed. Using unnecessary absolute-value symbols in a *final answer* is a tip-off that the student doesn't fully understand what he/she is writing, and doesn't understand why there are absolute-value symbols in "$\int \frac{1}{x}dx = \ln|x| + C$." It's fine to use unnecessary absolute-value symbols in an *initial or intermediate* step, but not in a final answer).

6. In this problem it is not possible to solve for y explicitly in terms of x, except for the constant solutions. There are several equivalent ways for writing the general solution in implicit form, two of which are:

(i) $\left\{y^2(1-y) = Ce^{x^2}(1+y) \mid C \in \mathbf{R}\right\}$ and $\{y = -1\}$.

(ii) $\left\{e^{x^2}(1+y) = Cy^2(1-y) \mid C \in \mathbf{R}\right\}$ and $\{y = 0\}$ and $\{y = 1\}$.

7. (a) $x = 2$ (remember that this means the constant *function*, $x(t) = 2$) ; domain $(-\infty, \infty)$.

(b) $x = \frac{2(3-e^{4t})}{3+e^{4t}}$ (this can be written different ways, e.g. $x = 2\frac{1-\frac{1}{3}e^{4t})}{1+\frac{1}{3}e^{4t}}$); domain $(-\infty, \infty)$.

(c) $x = -2$; domain $(-\infty, \infty)$.

(d) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\frac{1}{4}\ln 5, \infty)$.

(e) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\infty, -\frac{1}{4}\ln 5)$.

8. $y = -(3 - 2\sqrt{1+x^2})^{-1/2}$; domain $(-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2})$.

9. (a) $u = e^t(1 - 2t^{-1} + 2t^{-2}) + Ct^{-2}$.

(b) $y = (x\ln x - x + C)\sec x$.

(c) $y = \frac{1}{2}x^3[(x^2+1)\tan^{-1}x - x] + \left\{\begin{array}{ll} C_1x^3, & x \geq 0, \\ C_2x^3, & x < 0. \end{array}\right.$