

Notes on some book problems

Section 1.2

1, 14, 15, 17, 18, 20–22. Correct the wording of #1a by doing either of the following: (i) replace “ $\phi(x) = x^2$ ” by “ $y = x^2$ ”, or (ii) replace the phrase “ $\phi(x) = x^2$ is an explicit solution” by “the function ϕ defined by $\phi(x) = x^2$ is a solution”. Correct the wordings of 1bc and the other indicated problems analogously.

3–8. In #3, “the given function” means “the function y defined by the equation $y(x) = \sin x + x^2$.” A similar comment applies to the rest of the problems in this group.

15. It is not correct to say that the one-parameter family of functions ϕ_c defined by $\phi_c(x) = \frac{1}{1-ce^x}$, where c is an arbitrary constant, is a one-parameter family of solutions of the given DE. (Many or most textbooks make this mistake, as I did myself for the first 15 years or so that I taught differential equations.) For $c \leq 0$, the formula for ϕ_c does indeed yield a (single, maximal) solution of the DE, with domain $(-\infty, \infty)$. But for $c > 0$, the formula for ϕ_c corresponds to *two* maximal solutions, one on the interval $(-\infty, \ln \frac{1}{c})$ and one on the interval $(\ln \frac{1}{c}, \infty)$. By contrast, the family of functions in problem 17 is a *true* one-parameter family of solutions to the DE in that problem. (See my notes for what I mean by “maximal solution”.)