Answers to some of the non-book problems

2. (a) $u = e^t(1 - 2t^{-1} + 2t^{-2}) + Ct^{-2}$.
   (b) $y = (x \ln x - x + C) \sec x$.
   (c) $y = \frac{1}{2}x^3[(x^2 + 1) \arctan x - x] + \left\{ \begin{array}{ll} C_1 x^3, & x \geq 0, \\ C_2 x^3, & x < 0. \end{array} \right.$

3. $\{y(\ln y - 1) = -x \cos x + \sin x + C\}$. You can't solve explicitly for $y(x)$ in this example.

Writing "$y \ln y - y$" is just as good as writing "$y(\ln y - 1)$"; neither is clearly a simpler expression than the other.

I would not penalize you for omitting the curly braces in answers like this.

4. $\left\{e^{2y}(\frac{1}{2}y - \frac{1}{4}) = x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C\right\}$. You can't solve explicitly for $y(x)$ in this example.

I might penalize you a *small* amount for writing "$\ln|1+x^2|$" instead of $\ln(1+x^2)$ in a final answer, since this is an instance of failing to simplify. (Absolute-value symbols around a positive quantity are not needed. Using unnecessary absolute-value symbols in a *final answer* is a tip-off that the student doesn't fully understand what he/she is writing, and doesn't understand why there are absolute-value symbols in "$\int \frac{1}{x} dx = \ln|x| + C$." It's fine to use unnecessary absolute-value symbols in an *initial or intermediate* step, but not in a final answer).

5. In this problem it is again not possible to solve for y explicitly in terms of x, except for the three constant solutions ($y = 0, y = 1$, and $y = -1$). There are several equivalent ways for writing the general solution in implicit form, three of which are:

(i) $\left\{e^{-2/y}(1 + y) = Ce^{x^2}(1 - y) \mid C \in \mathbf{R},\ C \neq 0\right\}$ and $\{y = 0\}$ and $\{y = 1\}$ and $\{y = -1\}$.

In (i), it would not be terrible to omit the "$C \neq 0$ after "$C \in \mathbf{R}$"; the minor redundancy (accounting for the $y = -1$ solution twice [see (ii) below]) would not be a crime.

(ii) $\left\{e^{-2/y}(1 + y) = Ce^{x^2}(1 - y) \mid C \in \mathbf{R}\right\}$ and $\{y = 0\}$ and $\{y = 1\}$.

In (ii), the constant solution $y = -1$ has been absorbed, in implicit form, into the one-parameter family of equations labeled by C: when $C = 0$ the equation in this family is an implicit form of "$y = -1$." Each of (i) and (ii) has an advantage (and a disadvantage) relative to the other: The advantage of (i), relative to (ii), is that all the constant solutions are shown explicitly and treated symmetrically. The advantage of (ii), relative to (i), is that it involves less writing; however, this answer could potentially lead a reader to think, **mistakenly**, that the constant solution $y = -1$ is more special than the other two, describable as one member of a family of solutions. See "The myopic eye of the beholder" in my notes.

(iii) $\left\{e^{x^2}(1-y) = Ce^{-2/y}(1+y) \mid C \in \mathbf{R}\right\}$ and $\{y=0\}$ and $\{y=-1\}$.

Comments similar to those after (ii) apply; in (iii), the constant solution $y=1$ is accounted for, in implicit form, in the one-parameter family of equations labeled by (this answer's) C.

7. $y = -(3-2\sqrt{1+x^2})^{-1/2}$; domain $(-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2})$.

8. (a) $x = 2$ (remember that this means the constant *function*, $x(t) = 2$) ; domain $(-\infty, \infty)$.

(b) $x = \frac{2(3-e^{4t})}{3+e^{4t}}$ (this can be written different ways, e.g. $x = 2\frac{1-\frac{1}{3}e^{4t})}{1+\frac{1}{3}e^{4t}}$); domain $(-\infty, \infty)$.

(c) $x = -2$; domain $(-\infty, \infty)$.

(d) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\frac{1}{4}\ln 5, \infty)$.

(e) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\infty, -\frac{1}{4}\ln 5)$.

10. (a) $t^2/2 + \ln|t| = \frac{1}{2}\ln(s^2+1) + \arctan s + C$, and $t \equiv 0$

(b) $uv - u^2 - u + v^3/3 - 4u = C$

(c) $r = -\frac{7}{6}\theta + \frac{7}{3} + Ce^{-\theta/2}$

(d) The solutions that students would be expected to find are $w = e^x(1-2x^{-1}+2x^{-2})+Cx^{-2}$ and $x \equiv 0$. There is actually one additional solution that students would usually not be expected to find without guidance (at least not on an exam):

$$w = \begin{cases} e^x(1-2x^{-1}+2x^{-2}) - 2x^{-2} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(e) $3x^2 - x + xy + y^3 = 12$

(f) $y^4 + x^4/4 + x^2/2 = C$