Nonbook problems

1. Verify that for any nonzero constant b, the function $f(x) = \frac{1}{b}\cosh(bx)$ satisfies the differential equation

$$\frac{d^2y}{dx^2} - b\sqrt{1 + (\frac{dy}{dx})^2} = 0.$$

(Recall that the function "cosh" is defined by $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$.)

2. Verify that on the interval $-2 < x < 2$, the two continuous functions $y(x) = \sqrt{4 - x^2}$ and $y(x) = -\sqrt{4 - x^2}$, obtainable from the implicit solution $x^2 + y^2 = 4$ of the differential equation $x + y\ dy/dx = 0$, are (explicit) solutions of this differential equation.

3. Show that if a differential operator L is linear, then for all $n \geq 1$, all constants $c_1, c_2, \ldots, c_n$, and all functions $f_1, f_2, \ldots f_n$ (differentiable enough times for L of each function to be defined),

$$L[c_1f_1 + c_2f_2 + \ldots + c_nf_n] = c_1L[f_1] + c_2L[f_2] + \ldots + c_nL[f_n].$$