MAP 2302—Spring 2010—Section 3146
COMMENTS ON FIRST EXAM

Partial Credit Policy. In general, to get more than a point or two of partial credit you have to make *significant* progress towards a *correct* solution of *the question that was asked.*

General comments on the exam. Preparation for an exam should always include at least the following three components:

- reviewing the homework;
- reviewing the reading; and
- reviewing your class notes (especially any material not covered adequately in the reading or homework).

If you're given old exams to look at, it's certainly important to do so, but *that won't make up for missing any of the other components.* Remember what the syllabus states: "Unless I say otherwise, you are responsible for knowing any material I cover in class, any subject covered in homework, and all the material in the textbook chapters we are studying."

If you haven't kept up with your work in the weeks before an exam, it will be impossible the day or two or three before the exam to do all the studying that you need. I can't emphasize strongly enough the importance of doing your homework on time, seriously attempting *every* problem, and *carefully* re-reading (and preferably rewriting) your notes as soon as possible after class. If there's something you don't understand well enough to write down in sentences that another person would understand, then you should come to my next office hour with your question(s).

Please learn from your experience on the first midterm how you could have prepared more effectively for it. The comments below are meant to help you find which parts of your preparation were most lacking.

Comments on specific problems.

Problem 4. This is a separable equation. Part (a) is *exactly* HW problem 2.2/23, with the letter "x" instead of "t". If you had trouble with the y-integral when you did the homework problem, a good thing to do at the time would have been to make a note to yourself to review that integral when preparing for the exam.

For part (b), look at your notes from class on separable equations. More than once, we saw that the solutions of a separable equation $\frac{dy}{dx} = g(x)p(y)$ fall into two classes: (i) the non-constant solutions, which are exactly the ones that separation of variables finds, and (ii) the constant solutions (of which there sometimes are none) which separation of variables *never* finds. In the exam problem, $\cos^2 y$ is 0 whenever y is an odd multiple of $\pi/2$. So the DE has constant solutions of the form $y(x) = (n + \frac{1}{2})\pi$ (n an arbitrary integer). The solution to the IVP in part (b) is just the constant function $y(x) = \frac{\pi}{2}$. This solution cannot be found by separating variables.

For students who gave the answer that "There is no solution" (which was better than giving a *wrong* solution and not knowing it was wrong), note that the function of x and y on the right-hand side of the given DE is continuous and differentiable on the whole xy plane, so the Fundamental Existence/Uniqueness Theorem guarantees that *every* initial-value problem—including the one in part (b)—has a (unique) solution. Theorems matter! That's why you're taught them. If your exam-prep had included understanding what this theorem is saying, and reviewing all the times this theorem's importance has been emphasized in class, you'd have been clued in that the answer to part (b) *could not possibly be* "There is no solution."

Problem 5. This is another separable equation, as you see by rewriting it as $\frac{dy}{dx} = \frac{2}{x}(y+y^2)$. Once you separate variables[1], the y-integral you need to do is very similar to the x-integral in HW problem 2.2/13, which I did in class. Both are done with partial fractions.

Among those students who did the integral correctly and used the IC correctly, most got as far as the implicit solution $\frac{y}{y+1} = \frac{1}{3}x^2$, but for some reason stopped there. This equation is *easily* solved explicitly for $y(x)$. If you don't see how, look at your notes for how I got an explicit solution for 2.2/13. Without the explicit form of the solution, there is no way (in this example) to find the domain of the solution. With the explicit form, finding the domain $(-\sqrt{3} < x < \sqrt{3})$ is easy.

Problem 6. This is just an IVP-version of HW problem 2.4/32c. The final answer is $y = -\sqrt{x^2+4}$.

I decided to treat this problem as Extra Credit when I made up the grading scale, since I did not go over the topic of orthogonal-trajectories in class, and this topic has some conceptual and mechanical subtleties, and you saw only two examples in homework (the other one being 2.4/32b). But that does not excuse not doing, or at least attempting, these problems when they were assigned, and seeing me in my office ASAP if you had difficulties with them.

Problem 7. This is essentially a shortened version of HW problem 2.3/35b (shortened by giving you the initial conditions directly, instead of having you get them by solving another IVP first). I did the HW problem—parts (a) and (b)—in class on the Wednesday before the exam, in response to a question. In the exam problem the volume of water was *in*creasing at a rate of 1 L/min, whereas in the HW problem the volume of water was *de*creasing at a rate of 1 L/min, but the techniques for setting up and solving the DEs are essentially the same in both cases. If you had any difficulty doing the HW problem, then certainly your exam-prep should have included studying the way I solved the problem in class.

[1] The only constant solutions of this DE are $y(x) = 0$ and $y(x) = -1$, which do not satisfy the IC, so the solution of the given IVP is one of the solutions found by separating variables.