Answers to some of the non-book problems

3. $\{y(\ln y - 1) = -x\cos x + \sin x + C\}$. You can't solve explicitly for $y(x)$ in this example.

Writing "$y\ln y - y$" is just as good as writing "$y(\ln y - 1)$"; neither is clearly a simpler expression than the other.

I would not penalize you for omitting the curly braces in answers like this.

4. $\left\{e^{2y}(\frac{1}{2}y - \frac{1}{4}) = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C\right\}$. You can't solve explicitly for $y(x)$ in this example.

I might penalize you a *small* amount for writing "$\ln|1+x^2|$" instead of $\ln(1+x^2)$ in a final answer, since this is an instance of failing to simplify. (Absolute-value symbols around a positive quantity are not needed. Using unnecessary absolute-value symbols in a *final answer* is a tip-off that the student doesn't fully understand what he/she is writing, and doesn't understand why there are absolute-value symbols in "$\int \frac{1}{x}dx = \ln|x| + C$." It's fine to use unnecessary absolute-value symbols in an *initial or intermediate* step, but not in a final answer).

6. In this problem it is not possible to solve for y explicitly in terms of x, except for the constant solutions. There are several equivalent ways for writing the general solution in implicit form, two of which are:

(i) $\left\{y^2(1-y) = Ce^{x^2}(1+y) \mid C \in \mathbf{R}\right\}$ and $\{y = -1\}$.

(ii) $\left\{e^{x^2}(1+y) = Cy^2(1-y) \mid C \in \mathbf{R}\right\}$ and $\{y = 0\}$ and $\{y = 1\}$.

In a family of solutions, *purely additive* constants are understood to be arbitrary (i.e. they can assume any real value), so saying "$C \in \mathbf{R}$" is not necessary in problems 3 and 4 above (though it would not have be wrong to include this). In general, non-additive constants in families of solutions potentially have restrictions on the values they can assume. So for non-additive constants, e.g. the multiplicative constants in (i) and (ii), you should say whether there are any restrictions, and if so, what they are—provided it's algebraically possible to figure this out. In the method that produces answer (i) and (ii) (which are derived by simplifying "$\left\{\ln\left(y^2\left|\frac{1-y}{1+y}\right|\right) = x^2 + C\right\}$ and $\{y=0\}$ and $\{y=1\}$ and $\{y=-1\}$") it *is* possible to deduce that there are no restrictions on C, so this fact has been stated in those answers.

7. (a) $x = 2$ (remember that this means the constant *function*, $x(t) = 2$) ; domain $(-\infty, \infty)$.

(b) $x = \frac{2(3-e^{4t})}{3+e^{4t}}$ (this can be written different ways, e.g. $x = 2\frac{1-\frac{1}{3}e^{4t})}{1+\frac{1}{3}e^{4t}}$); domain $(-\infty, \infty)$.

(c) $x = -2$; domain $(-\infty, \infty)$.

(d) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\frac{1}{4}\ln 5, \infty)$.

(e) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\infty, -\frac{1}{4}\ln 5)$.

8. (a) $u = e^t(1 - 2t^{-1} + 2t^{-2}) + Ct^{-2}$.

(b) $y = (x \ln x - x + C) \sec x$.

(c) $y = \frac{1}{2}x^3[(x^2+1)\tan^{-1} x - x] + \begin{cases} C_1 x^3, & x \geq 0, \\ C_2 x^3, & x < 0. \end{cases}$