Answers to some of the non-book problems

2. (a) $u = e^t(1 - 2t^{-1} + 2t^{-2}) + Ct^{-2}$.
   (b) $y = (x\ln x - x + C)\sec x$.
   (c) $y = \frac{1}{2}x^3[(x^2+1)\arctan x - x] + \begin{cases} C_1x^3, & x \geq 0, \\ C_2x^3, & x < 0. \end{cases}$

3. $\{y(\ln y - 1) = -x\cos x + \sin x + C\}$. You can't solve explicitly for $y(x)$ in this example.

Writing "$y\ln y - y$" is just as good as writing "$y(\ln y - 1)$"; neither is clearly a simpler expression than the other.

I would not penalize you for omitting the curly braces in answers like this.

4. $\left\{e^{2y}(\frac{1}{2}y - \frac{1}{4}) = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C\right\}$. You can't solve explicitly for $y(x)$ in this example.

I might penalize you a *small* amount for writing "$\ln|1+x^2|$" instead of $\ln(1+x^2)$ in a final answer, since this is an instance of failing to simplify. (Absolute-value symbols around a positive quantity are not needed. Using unnecessary absolute-value symbols in a *final answer* is a tip-off that the student doesn't fully understand what he/she is writing, and doesn't understand why there are absolute-value symbols in "$\int \frac{1}{x}dx = \ln|x| + C$." It's fine to use unnecessary absolute-value symbols in an *initial or intermediate* step, but not in a final answer).

5. In this problem it is not possible to solve for y explicitly in terms of x, except for the constant solutions. There are several equivalent ways for writing the general solution in implicit form, two of which are:

   (i) $\left\{e^{-2/y}(1+y) = Ce^{x^2}(1-y) \mid C \in \mathbf{R}\right\}$ and $\{y = 0\}$ and $\{y = 1\}$.
   (ii) $\left\{e^{x^2}(1-y) = Ce^{-2/y}(1+y) \mid C \in \mathbf{R}\right\}$ and $\{y = 0\}$ and $\{y = -1\}$.

7. $y = -(3 - 2\sqrt{1+x^2})^{-1/2}$; domain $(-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2})$.

8. (a) $x = 2$ (remember that this means the constant *function*, $x(t) = 2$) ; domain $(-\infty, \infty)$.

   (b) $x = \frac{2(3-e^{4t})}{3+e^{4t}}$ (this can be written different ways, e.g. $x = 2\frac{1-\frac{1}{3}e^{4t}}{1+\frac{1}{3}e^{4t}}$); domain $(-\infty, \infty)$.

   (c) $x = -2$; domain $(-\infty, \infty)$.

   (d) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\frac{1}{4}\ln 5, \infty)$.

   (e) $x = \frac{2(1+5e^{4t})}{1-5e^{4t}}$; domain $(-\infty, -\frac{1}{4}\ln 5)$.

10. (a) $t^2/2 + \ln|t| = \frac{1}{2}\ln(s^2+1) + \arctan s + C$, and $t \equiv 0$

(b) $uv - u^2 - u + v^3/3 - 4u = C$

(c) $r = -\frac{7}{6}\theta + \frac{7}{3} + Ce^{-\theta/2}$

(d) The solutions that students would be expected to find are $w = e^x(1-2x^{-1}+2x^{-2})+Cx^{-2}$ and $x \equiv 0$. There is actually one additional solution that students would usually not be expected to find without guidance (at least not on an exam):

$$w = \begin{cases} e^x(1 - 2x^{-1} + 2x^{-2}) - 2x^{-2} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(e) $3x^2 - x + xy + y^3 = 12$

(f) $y^4 + x^4/4 + x^2/2 = C$