MAA 4211, Fall 2001—Homework # 6 non-book problems

Hand in both problems.

B1. Let d, d' be equivalent metrics (as defined on the last exam) on a nonempty set E.
(a) Prove that (E, d) is complete iff (E, d') is complete.
(b) Prove that (E, d) is compact iff (E, d') is compact.

B2. (a) Let $S \subset \mathbf{E}^n$ and let p be a point of $\mathbf{E}^n$ not lying in S. Suppose that there is a point $q \in S$ that, among all points in S, minimizes distance to p. Prove that q lies in the boundary of S. (In the terminology of the last exam, problem 6, the hypothesis here is that $dist(p, S) = d(p, q)$—i.e. that the infimum defining $dist(p, S)$ is actually a minimum, achieved at q. If you do not know the distinction between a minimum and an infimum [=g.l.b. when finite], review your notes.)

(b) If we replace $\mathbf{E}^n$ in part (a) by an arbitrary metric space, is the conclusion still true? (If yes, give a proof; if no, give a counterexample.)