MAA 4211, Fall 2016—Assignment 2's non-book problems

B1. Prove that every subset of a countable set is countable.

B2. Let X, Y be sets, with X countable. Prove that if there exists a surjective map $X \to Y$, then Y is countable.

B3. Prove that every infinite set has a countably infinite subset.

B4. Prove that a nonempty set X is countable if and only if there exists a surjective map $\mathbf{N} \to X$.

B5. Prove that a countable union of countable sets is countable; i.e., if $\{A_i\}_{i \in I}$ is a collection of sets, indexed by $I \subset \mathbf{N}$, with each A_i countable, then $\bigcup_{i \in I} A_i$ is countable. *Hints*: (i) Show that it suffices to prove this for the case $I = \mathbf{N}$. (ii) By Problem B4, for each $i \in I$ either $A_i = \emptyset$ or there is a surjective map $f_i : \mathbf{N} \to A_i$. Use these maps to produce a surjective map $\mathbf{N} \times \mathbf{N} \to \bigcup_{i \in \mathbf{N}} A_i$, and then use earlier results to conclude that $\bigcup_{i \in \mathbf{N}} A_i$ is countable.