Proof-writing quiz

Consider the following theorem.

Zorp-Floog Theorem. Every zorp is a floog.

1. Which of the following is/are equivalent to the statement of the Zorp-Floog theorem?
   a. Every floog is a zorp.
   b. Some floogs are zorps.
   c. Some zorps are floogs.
   d. If X is a zorp, then X is a floog.
   e. If X is a floog, then X is a zorp.
   f. If X is not a zorp, then X is not a floog.
   g. If X is not a floog, then X is not a zorp.

2. Which of the following are good ways to start a valid proof of the Zorp-Floog theorem? Which are definitely not starts of a valid proof if the apparent train of thought (proving something about X or, in choice 'i', about X and/or Y) is continued? Which contain a statement/phrase or statements/phrases that, while not yet invalidating the proof, are extraneous to a valid proof?
   a. "If X is a zorp, then ..."
   b. "If X is a floog, then ..."
   c. "X is a floog if ..."
   d. "Assume that X is a zorp."
   e. "Assume that X is a floog."
   f. "Let X be a zorp."
   g. "Let X be a floog."
   h. "Since X is a zorp, ..."
   i. "Let X be a zorp and let Y be a floog."
   j. "Let X be a zorp. If X is a floog, then ..."

3. Suppose that, in the course of writing a proof (of either the Zorp-Floog Theorem or any other) you write the following:

$$\text{"Let } A = \{x \in \mathbf{R} \mid x < 2\}."$$

Which (one or more) of the following could you properly say next?
   a. "Since $x < 2$, ..."
   b. "Since $y < 2$, ..."
   c. "If $x < 2$, ..."
   d. "If $y \in A$, then ...."
   e. "$x < 2$, so ...."
   f. "Let $x \in A$. Then $x < 2$, so ...."

g. "Let $y \in \mathbf{R}$ and assume $y < 2$. Then $y \in A$, so ...."

Note: In choice 'd', the portion before the comma is read "If y *is* in A", not "If y in A". Similarly, in choice 'f', the first sentence is read "Let x *be* in A" or "Let x *be* an element of A, not "Let x in A."

4. Again suppose that, in the course of writing a proof, you write "Let $A = \{x \in \mathbf{R} \mid x < 2\}$." Which (one or more) of the following could you properly say next?
   a. "If $x \in A$, then $3x < 6$. Let $y = 3x$. Thus $y < 6$."
   b. "Assume that $x \in A$. Let $y = 3x$. Then $y < 6$."
   c. "Let $x \in A$. Let $y = 3x$. Then $y < 6$."
   d. "For every $x \in A$, let $y = 3x$. Then $y < 6$."

Note: In choice 'a', it is irrelevant that the first word of the last sentence is "Thus" as opposed to "Then", "Therefore", "Hence", etc.

5. In written mathematics, which (one or more) of the following indicate(s) the end of a sentence?

   a. A large space after a word.

   b. A large space after a word, with the next word capitalized.

   c. The last word or symbol on a line.

   d. The writer's knowledge that were he/she reading aloud what he/she's written, he/she would verbally pause the way one does at the end of a sentence.

   e. Some combination of the above.

   f. A period.

6. Consider the following bit of writing:

   If x equals 10 and -7 is greater than y or 7 is less than y implies y^2 is greater than 49 because y is less than zero in the first case and shows that x does not equal y Because x^2 equals 100 which is greater than 49 Therefore we are done

True or false:

   a. It's the reader's responsibility to figure out what the writer meant, after which the reader can figure out whether what the writer meant was correct.

   b. Regardless of the correctness of the argument the writer had in mind, the passage above is gibberish.

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7. Consider the following modified version of the passage in the previous question:

$$x = 10$$
$$-7 > y \quad \text{or} \quad 7 < y$$
$$y^2 > 49$$
$$x \neq y$$
$$\text{Because } x^2 = 100 > 49$$

True or false:

a. Since it is possible to figure out a correct argument that the writer appears to have had in mind, the passage above (in question 7) is an acceptable proof that $x \neq y$.

b. The passage above (in question 7) is gibberish.