MAA 4212, Spring 2009—Homework # 1 non-book problems

Hand in only A2.

A1. Let $a, b \in \mathbf{R}, a < b$, and let $f : [a, b] \to \mathbf{R}$. Prove that if f is integrable on $[a, b]$, then for any sequence $(\mathcal{P}_n, T_n)$ of pointed partitions for which $\|\mathcal{P}_n\| \to 0$ as $n \to \infty$,

$$\lim_{n\to\infty} S(f; \mathcal{P}_n, T_n) = \int_a^b f.$$

(Hence the integral can be evaluated by taking such a limit, *if you know ahead of time that f is integrable.*)

A2 (formerly called A1). For any real-valued function f, the *positive part of* f, denoted f_+, and *negative part of* f, denoted f_-, are defined by $f_+(x) = \max\{f(x), 0\}$ and $f_-(x) = -\min\{f(x), 0\}$. (Thus both f_+ and f_- are non-negative, and $f = f_+ - f_-$ [why?].)

Let $a, b \in \mathbf{R}, a < b$, and let $f : [a, b] \to \mathbf{R}$. Prove that if f is integrable on $[a, b]$, then so are f_+ and f_-.