MAA 4212

Notational and Terminological Differences in Source Materials

There are several objects in mathematics for which there are two mutually inconsistent name-conventions (or definition-conventions), each followed by a large segment of the mathematical community. One of these is the definition of "natural numbers" (specifically, whether 0 is included in this set); another is the definition of "countable set".[1] There are other examples as well. There are also objects for which all mathematicians have the same (or equivalent) *definitions*, but for which they use different *notation*. One example is the notation for the complement of a subset.

If you take MAA 4211 and MAA 4212 from two different professors, there is a very good chance that your MAA 4212 professor will not adhere to the same convention or notation as your MAA 4211 professor, either in the examples above or in others.

Below (starting on the next page) are some differences in terminology and notation that students may encounter when comparing Rosenlicht's textbook, Dr. Groisser's handouts and work in class, and Dr. McCullough's notes (the notes you used if you took Dr. Jury's or Dr. Vatter's section of MAA 4211 in Fall 2016). When Rosenlicht has terminology or notation for some object, Dr. Groisser generally tries to adhere to Rosenlicht's when teaching MAA 4211-12, but there are a few exceptions.

One general note of explanation concerning notation for various sets of numbers: Historically, most math books and articles used boldface notation for the set of real numbers ($\mathbf{R}$), the set of complex numbers ($\mathbf{C}$), etc. But boldface doesn't work on the blackboard (or when taking notes), so people developed the symbols $\mathbb{R}, \mathbb{C}$, etc., called "blackboard bold", purely for handwritten use, while keeping the boldface notation in their published materials. In recent decades, "blackboard bold" has taken on a life as its own font in printed materials, rather than just as a substitute for actual boldface. Dr. Groisser's personal preference is the usage he grew up with: $\mathbf{R}$ in printed materials, $\mathbb{R}$ at the blackboard (etc. for several other objects).

[1] Two of many textbooks that use the same conventions as Dr. Groisser for "countable set" are Edwin Hewitt and Karl Stromberg, *Real and Abstract Analysis*, Springer-Verlag 1969, and James R. Munkres, *Topology*, 2nd ed., Prentice Hall 2000. One of many textbooks that use a different convention for "countable set" (there are more than two conventions for this concept) is Walter Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill 1976. Two of many textbooks that use the convention that "natural number" means "positive integer" (i.e. that 0 is not a natural number) are Avner Friedman, *Advanced Calculus*, Holt, Rinehart & Winston 1971 (Dover reprint 2007), and the textbook for this course: Maxwell Rosenlicht, *Introduction to Analysis*, Scott, Foresman, and Co. 1968 (Dover reprint 1986). Rudin, *op. cit.*, is one of many textbooks that uses the convention in which 0 *is* a natural number. Hewitt and Stromberg, *op. cit.*, circumvents the convention-inconsistency for "natural number" by avoiding all use of the term "natural number" (a choice made by many other authors as well), but still uses the notation N for the set of positive integers.

Object	Rosenlicht	Groisser	McCullough notes
{natural numbers} (definition)	$\{1, 2, 3, \dots\}$	$\{1, 2, 3, \dots\}$	$\{0, 1, 2, 3, \dots\}$
{natural numbers} (symbol)	no symbol	$\mathbf{N}$ in printed materials, $\mathbb{N}$ on blackboard	$\mathbb{N}$
{integers}	no symbol	$\mathbf{Z}$ in printed materials, $\mathbb{Z}$ on blackboard	$\mathbb{Z}$
{positive integers} (symbol)	no symbol	$\mathbf{N}$ in printed materials, $\mathbb{N}$ on blackboard	$\mathbb{N}^+$
{rational numbers}	no symbol	$\mathbf{Q}$ in printed materials, $\mathbb{Q}$ on blackboard	$\mathbb{Q}$
{real numbers}	$\mathbf{R}$	$\mathbf{R}$ in printed materials, $\mathbb{R}$ on blackboard	$\mathbb{R}$
{complex numbers}	$\mathbf{C}$	$\mathbf{C}$ in printed materials, $\mathbb{C}$ on blackboard	$\mathbb{C}$
complement of a subset X (of a specified larger set)	$\mathcal{C}X$ (this is as close as I can come to Rosenlicht's "curly C")	$\mathcal{C}X$	$\tilde{X}$ or $\widetilde{X}$
empty set	circle with a forward-slash through it	same as Rosenlicht on blackboard; $\emptyset$ in printed materials. (Unfortunately, "$\emptyset$" looks very similar to computer scientists' "zero".)	$\emptyset$
set difference (= relative complement) "X minus Y"	$X - Y$	$X \setminus Y$	$X \setminus Y$
range (= image) of function $f : X \to Y$	$f(X)$	img(f) or image(f) or $f(X)$	rg(f)

Object	Rosenlicht	Groisser	McCullough notes
certain adjectives for sets		countable	at most countable
certain adjectives for sets		countably infinite	countable
open ball of radius r centered at p (in a metric space)	no specific notation	$B_r(p)$	$N_r(p)$
interior of a set S (in a metric space)	no specific notation	$\text{Int}(S)$ or $\overset{\circ}{S}$ or S°	S°
terminology for sequences	sequence of points in a metric space (or set) X	sequence *in* X	sequence *from* X
notation for sequences	$p_1, p_2, p_3, \ldots$	$(p_n)_{n=n_0}^\infty$ or (p_n)	$(p_n)_{n=n_0}^\infty$ or $(p_n)_n$ or (p_n)
accumulation point of a sequence	no name for this	accumulation point	no name for this
limit point of a subset of a metric space	cluster point	cluster point	limit point