Non-book problem for Assignment 7

NB 7.1. Use the Corollary on p. 201 to show the following: If G and $\overline{G}$ are finite groups, and $\phi : G \to \overline{G}$ is a homomorphism, then for every $g \in G$ the order of $\phi(g)$ divides both $|G|$ and $|\overline{G}|$.

NB 7.2. Let G be a group. Assume that H and K are subgroups of G satisfying the following conditions:

  (i) $HK = G$.

  (ii) $H \cap K = \{e\}$.

  (iii) $K \triangleleft G$.

(Recall that in the definition of internal direct product of two subgroups, *both* of these subgroups were assumed to be normal [in addition to satifisfying conditions (i) and (ii) above]. Here only one of the subgroups is assumed normal.) Prove that G/K is isomorphic to H.