MTG 6256, Fall 2004: Non-book Problem 3

Let $\gamma : \mathbf{R} \to \mathbf{R}^3$ be a unit-speed curve with constant nonzero curvature κ and constant torsion τ. Let $\gamma(0) = p_0$ ant let $T_0, N_0, B_0 \in \mathbf{R}^3$ be the images of the Frenet-frame vectors $T(0), B(0), N(0) \in T_{p_0}\mathbf{R}^3$. Using the matrix-exponential method discussed in class, show that γ is a helix or a circle by finding numbers $a > 0, b \in \mathbf{R}$, a point $p_1 \in \mathbf{R}^3$, and a right-handed orthonormal basis $\{E_1, E_2, E_3\}$ of $\mathbf{R}^3$, all expressed in terms of $T_0, N_0, B_0, \kappa, \tau$, and p_0, such that

$$\gamma(s) = p_1 + a\cos(\frac{s}{c})E_1 + a\sin(\frac{s}{c})E_2 + b(\frac{s}{c})E_3,$$

where $c = \sqrt{a^2 + b^2}$.