

MTG 6256, Fall 2004: Non-book Problem 5

(a) Complete the example started in class: for the spherical-coordinate orthonormal frame field $\{E_i\}$ constructed in class, express the dual 1-forms in terms of $d\rho$, $d\theta$, and $d\phi$.

(b) Using part (a) and non-book problem 4, express the $\{E_i\}$ in terms of the (non-orthonormal) frame-field $\{\frac{\partial}{\partial\rho}, \frac{\partial}{\partial\theta}, \frac{\partial}{\partial\phi}\}$.

(c) Compute the connection forms for the frame-field $\{E_i\}$. This is O'Neill's problem 2.7/4, but remember that O'Neill's ϕ is $\{\pi/2 - (\text{my } \phi)\}$, so your answers will look a little different from those in the book (but should agree up to this change-of-variable))

(d) Compute all nine covariant derivatives $\nabla_{E_i} E_j$. Explain why this part of the problem is made much easier by your having done part (c) first.