Exponential Review Sheet.

Below, all constants and variables are *real*.

Algebra.

- $e^A \cdot e^B = e^{A+B}$.

  Examples.

  $e^{2x} \cdot e^{3x} = e^{2x+3x} = e^{5x}$

  $e^{ax}e^{bx} = e^{(a+b)x}$

- $\frac{e^A}{e^B} = e^{A-B}$.
- $(e^A)^B = e^{AB}$.
- $(e^A)^B \neq e^A \cdot e^B$.
- $e^A + e^B$ usually cannot be simplified.
- $e^0 = 1$.
- $e^{-A} = \frac{1}{e^A}$
- e^r is not the same thing as e^{rx}. If r is a constant, so is e^r.

Limits

Below, a, b and n are constants.

- $\lim_{x\to\infty} e^x = \infty$
- $\lim_{x\to-\infty} e^x = 0$
- $\lim_{x\to\infty} e^{-x} = 0$
- $\lim_{x\to-\infty} e^{-x} = \infty$
- $\lim_{x\to\infty} e^{ax} = \begin{cases} \infty & \text{if } a > 0 \\ 0 & \text{if } a < 0 \\ 1 & \text{if } a = 0 \end{cases}$
- $\lim_{x\to\infty} x^n e^{-x} = 0$
- If $a \neq 0$ then for any n, $\lim_{x\to\infty} x^n e^{ax} = \begin{cases} \infty & \text{if } a > 0 \\ 0 & \text{if } a < 0 \end{cases}$

- If c_1, c_2 are nonzero constants, then $\lim_{x\to\infty}(c_1e^{a_1x} + c_2e^{a_2x}) = \lim_{x\to\infty} c_me^{a_mx}$, where a_m is the larger of a_1, a_2. I.e. when computing the limit you can just cross out the term with the smaller exponent. Note that this limit will be negative if c_m is negative. Example: $\lim_{x\to\infty}(7e^{3x} - 6e^{5x}) = \lim_{x\to\infty}(-6e^{5x}) = -\infty$. Even if c_1, c_2 are replaced by powers of x or by polynomials, the term $c_me^{a_mx}$ determines the limit. Similar rules apply to linear combinations $c_1e^{a_1x} + \ldots + c_ne^{a_nx}$ of more than two exponentials (with nonzero constants c_i): the term with the largest exponent (largest a_i) completely determines the limit as $x \to \infty$.

Below, let trig(x) denote either $\sin x$ or $\cos x$.

- $\lim_{x\to\infty} e^{-x}\text{trig}(x) = 0$.
- $\lim_{x\to\infty} x^ne^{-x}\text{trig}(x) = 0$.
- $\lim_{x\to\infty} e^{ax}\text{trig}(x) \begin{cases} \text{does not exist} & \text{if } a \geq 0 \\ = 0 & \text{if } a < 0 \end{cases}$

Derivatives and Integrals

Below, a is a constant.

- $\frac{d}{dx}e^x = e^x$, NOT xe^x or xe^{x-1}.
- $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$
- $\frac{d}{dx}e^{ax} = ae^{ax}$, NOT xe^{ax} or axe^{ax-1}.
- $\int e^{ax}dx = \frac{1}{a}e^{ax} + C$, if $a \neq 0$
- $\int e^{f(x)}dx \neq \frac{e^{f(x)}}{f'(x)} + C$ unless $f(x) = ax + b$ for some constants a, b.
- $\frac{d}{dx}(e^{ax}f(x)) = e^{ax}(af(x) + f'(x))$.

Exercises

A. Simplify the following.

1. $e^{-9x}e^{6x} + e^{4x}$
2. $(e^x)^2$
3. e^3e^{4x}

B. Compute the limits below.

1. $\lim_{t\to\infty} t^5e^{-t}\sin(56t)$.
2. $\lim_{t\to\infty} t^{-5}e^t\cos(56t)$.

C. Compute $\frac{d}{dx}(\frac{e^{f(x)}}{f'(x)})$, and explain why $\int e^{f(x)}dx \neq \frac{e^{f(x)}}{f'(x)} + C$ unless $f(x) = ax + b$ for some constants a, b.

D. Compute $f'(x)$ and $f''(x)$ for the functions below.

1. $f(x) = e^{6x}$
2. $f(x) = xe^x$
3. $f(x) = e^{2x}\cos(3x)$
4. $f(x) = xe^x \sin x$
5. $f(x) = e^{\sqrt{2}}$