Taking and Using Notes in a College Math Class

Since different teachers have different styles, perhaps I should have entitled this handout "Taking and Using Notes in *My* Math Class". Nonetheless I think that the suggestions below will stand you well in most of your math classes.

Allie Gator takes notes the following way. If the instructor writes something on the blackboard, Allie copies it down—every equation, every symbol, every word. If the instructor faces the class and says something but does not write it on the blackboard, Allie does not write it down.

Many math students take notes just the way Allie does. The moment the instructor puts down the chalk, the students put down their pens. However, ***this is almost the exact opposite of the right way to take notes.*** If Allie's method of taking notes is yours, then you're not getting the most out of your note-taking (or your class time).

Allie assumes that everything important will be written on the blackboard —necessarily with the instructor having his back to part or all of the class. This is **not** what a college math student should expect. To the extent possible, classroom interaction should be between student and teacher, not between teacher (or student) and blackboard. By the nature of the subject, this is more difficult for the instructor to arrange in a math class than in, say, an English class, and a math instructor will always have to write *some* of the important material on the blackboard. I use the blackboard to communicate information that can't effectively be communicated verbally, such as the steps involved in solving an equation. That doesn't mean that what's on the blackboard is the only important information I give you in class. If it were, I would spend all 50 minutes facing the blackboard.

Often I'll give you some of the most important information when I am *not* writing on the blackboard. In addition, when I *am* writing, I am also talking, and I may be saying something important *about* what I'm writing. **It is nearly impossible to take very detailed notes of blackboard work and pay attention to what's being said simultaneously.** When I solve problems at the blackboard, I usually proceed at a pace at which I think students should be able to follow *if they are paying attention to what*

I'm saying verbally, and are writing down only **important** *things.* A much higher proportion of what I'm *saying* is important than of what I'm writing. *Most* of what's said but not written on the blackboard should probably be represented in your notes—not necessarily every word, just every main idea or important fact, just as if you were taking notes in, say, a history class. And certainly when I *say* that something is important, it would be wise to believe me, and to write it down, whether or not I write it on the blackboard.

In contrast, *most* of what I put on the blackboard when I'm solving problems will be algebra that I do explicitly just to enable students to follow along—algebraic manipulations of the sort you've done hundreds of times and should be able to reproduce easily on your own. *If you try to copy down everything I write at the board, you will almost always be behind, racing to keep up with blackboard writing. You will barely be listening to the important things that I am saying, and you won't remember them or be able to reconstruct them later.* Remember that, just as with a class in any other subject, the purpose of taking notes in a math class is to enable yourself later to reconstruct arguments that you otherwise wouldn't remember in detail, and to make note of important facts, insights, or viewpoints that you were not aware of (or that you were aware of but whose importance you previously underestimated). The purpose is not to write algebraic details that you could have worked out on your own.

Example. Suppose that in the course of solving some problem, the instructor has to solve the equation

$$x = \frac{y-2}{y+3}$$

for y in terms of x. He might put the following on the blackboard:

$$\begin{array}{lrcl} & x & = & \dfrac{y-2}{y+3} \\ \Longrightarrow & x(y+3) & = & y-2, \\ \Longrightarrow & xy+3x & = & y-2, \\ \Longrightarrow & xy-y & = & -3x-2, \\ \Longrightarrow & y(x-1) & = & -(3x+2), \\ \Longrightarrow & y & = & -\dfrac{3x+2}{x-1}\ . \end{array}$$

Any student who's made it through pre-calculus should be able to get from the original equation to the final answer above on his or her own. Instead of occupying your brain with the details of writing down every character in every equation, you should be paying attention. What you could write in your notes is simply

$$\begin{array}{ccc} x & = & \dfrac{y-2}{y+3} \\ & \text{(algebra)} & \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ y & = & -\dfrac{3x+2}{x-1}, \end{array}$$

leaving enough space between the first line and the last that you can fill in the steps after class (which you would do to make sure that you really did know how to solve this equation). Or, if you don't think you'd have known to do the first step, but would have been able to proceed from there, you could write something like

$$\begin{array}{ccc} x & = & \dfrac{y-2}{y+3} \\ & \text{(clear fraction)} & \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ y & = & -\dfrac{3x+2}{x-1}, \end{array}$$

By intelligently limiting what you write, as above, you will be able to keep up better with what the instructor is doing. So, when your instructor is working an example, don't always copy every step, but only enough to enable yourself to reconstruct the work later (as soon as possible after class). Focus on things you wouldn't have thought to do on your own.

More generally, write down anything the instructor seems to emphasize or says is important. When the instructor turns away from the blackboard and faces the class, that's when you should get ready to write, not put your pen down. Write down concepts, however clear they may appear at the time the instructor says them. A good lecturer can make difficult concepts appear clear and easy. Later on, the train of thought can evaporate from your memory like last night's dream, no matter how attentive you were in class.

How you *use* your notes is as important as how you take them. Mathematics **cannot** be learned passively. You should review (and preferably, rewrite) your notes as soon as possible after each class, filling in all the gaps, and making a list of everything you don't understand, or of gaps you couldn't fill in. (However, if you *consistently* find yourself unable to fill in gaps in simple algebra or simple calculus steps, *this is a sign that the class may be above your head*, in which case taking overly detailed notes won't save you.)

The methods I've suggested above may require more in-class mental engagement and after-class work with your notes class than you are used to, but you will learn more and retain it much longer. Mathematics that is quickly and easily learned is quickly and easily forgotten.